Sample Question Paper-5

MATHEMATICS STANDARD

Class-X

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into 5 Sections A, B, C, D and E.*
3. *In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.*
4. *In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.*
5. *In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.*
6. *In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.*
7. *In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.*
9. *Draw neat and clean figures wherever required.*
10. *Take* $\pi = \frac{22}{7}$ *wherever required if not stated.*
11. *Use of a calculator is not allowed.*

Section – A

This section consists of 20 questions of 1 mark each.

1. In a game, a player must gather 20 flags positioned 5 m apart in a straight line.
The starting point is 10 m away from the first flag. The player starts from the starting point, collects the 20 flags and comes back to the starting point to complete one round.
What will be the total distance covered by a player upon completing one round?
(A) 105 m **(B)** 210 m **(C)** 220 m **(D)** 1,150 m [A]

2. The product of a non-zero rational and an irrational number is
(A) always irrational. **(B)** always rational. **(C)** rational or irrational. **(D)** one. [U]

3. For the following distribution: [U]

CI	0–5	6–11	12–17	18–23	24–29
f	26	20	30	16	32

The upper limit of the median class is
(A) 11:5. **(B)** 13.5. **(C)** 23.5. **(D)** 17.5.

4. The base radii of two circular cones of the same height are in the ratio 3:5. The ratio of their volumes are [R]
(A) 3:5 **(B)** 25:9 **(C)** 9:25 **(D)** 4:5

5. If the zeros of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and –3, then [A]
(A) $a = -7, b = -1$. **(B)** $a = 0, b = -6$. **(C)** $a = 5, b = -1$. **(D)** $a = 2, b = -6$.

6. If the lines given by the equations $2x + ky = 1$ and $3x - 5y = 7$ are parallel, then the value of k is
(A) $\frac{10}{3}$. **(B)** $\frac{5}{3}$. **(C)** $\frac{-10}{3}$. **(D)** $\frac{-5}{3}$. [E]

7. The value(s) of k, for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
(A) 4. **(B)** ±4. **(C)** –4. **(D)** 0. [R]

8. The centre of a circle whose endpoints of a diameter are (–6, 3) and (6, 4) is

(A) (8, –1). **(B)** (4, 7). **(C)** $\left(0, \frac{7}{2}\right)$. **(D)** $\left(4, \frac{7}{2}\right)$. [A]

9. In ΔABC and ΔDEF, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \frac{1}{2}DE$.

Then the two triangles are

(A) congruent but not similar. **(B)** similar but not congruent.

(C) neither congruent nor similar. **(D)** congruent as well as similar. [U]

10. If $\cos(40°+A) = \sin 30°$, then value of A is [E]

(A) 30°. **(B)** 60°. **(C)** 45°. **(D)** 20°.

11. From a point P which is at a distance of 13 cm from the centre O of a circle with a radius of 5 cm, the pair of tangents PQ and PR to the circle are drawn. The length of each tangent is [E]

(A) 5 cm. **(B)** 13 cm. **(C)** 12 cm. **(D)** 9 cm.

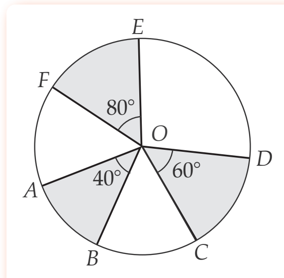
12. A shoe store owner is planning to stock up for the upcoming month. To make an informed decision, she reviews the sales data of various shoe sizes from the past six months. Which central tendency measure would help her in determining which shoe size to order the most of?

(A) Mean **(B)** Median **(C)** Mode **(D)** Any of the above [U]

13. In the given figure, three sectors of a circle with a radius of 7cm, making angles of 60°, 80° and 40° at the centre, are shaded. The area of the shaded region is [E]

(A) 77 cm^2. **(B)** 44 cm^2.

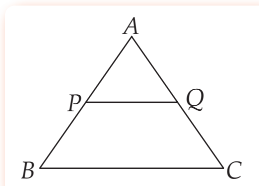
(C) 75 cm^2. **(D)** 154 cm^2.



14. In the given figure P and Q are points on the sides AB and AC, respectively, of ΔABC such that $AP = 3.5$ cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC. [A]

(A) 13.5 cm **(B)** 15 cm

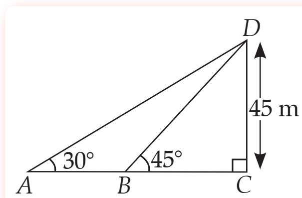
(C) 13 cm **(D)** 3 cm



15. In the given figure, what is the length of AB? [AI] [A]

(A) $45\sqrt{3}$ m **(B)** $\frac{45}{\sqrt{3}}$ m

(C) $45(\sqrt{3}-1)$ m **(D)** $45(\sqrt{3}+1)$ m



16. If $\sin 2A = \frac{1}{2}\tan^2 45°$ where A is an acute angle, then the value of A is [E]

(A) 60°. **(B)** 45°. **(C)** 30°. **(D)** 15°.

17. A library receives a shipment of a series of encyclopaedias. The shipment includes volumes 31–40. These encyclopaedias arrived in a box and are not ordered. One encyclopaedia is picked at random from the box without looking into it.

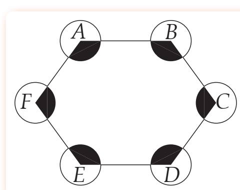
What is the probability that the volume of the encyclopaedias picked is a multiple of 2 or 5?

(A) $\frac{1}{10}$ **(B)** $\frac{5}{10}$ **(C)** $\frac{6}{10}$ **(D)** $\frac{7}{10}$ [U]

18. $ABCDEF$ is a hexagon with different vertices A, B, C, D, E and F as the centres of a circle with the same radius r are drawn. The area of shaded portion is [A]

(A) $3\pi r^2$. **(B)** $\frac{1}{3}\pi r^2$.

(C) πr^2. **(D)** $2\pi r^2$.



Directions: In the question numbers **19 and 20,** a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are correct, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The arithmetic mean of the weight of 12 workers in a factory is 64.25 kg. R

Weight (in kg)	60	63	66	69	72
No. of workers	4	3	2	2	1

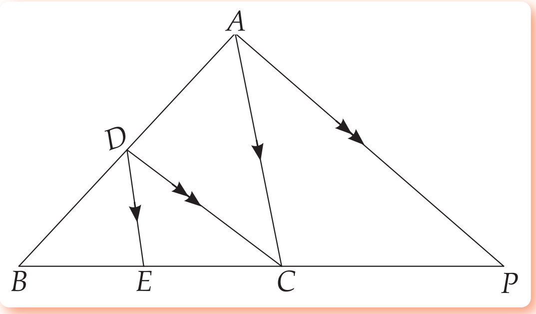
Reason (R): Arithmetic mean $= \dfrac{\sum f_i x_i}{\sum f_i}$.

20. **Assertion (A):** If two identical solid cubes of side 5 cm each are joined end to end, then the total surface area of the resulting cuboid is 300 cm^2.
Reason (R): Total surface of the cuboid $=2(lb + bh + hl)$. A

Section – B

This section consists of 5 questions of 2 marks each.

21. In the adjoining figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\dfrac{BE}{EC} = \dfrac{BC}{CP}$.



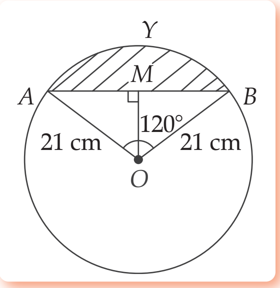
AI A

22. If $\cot\theta = \dfrac{7}{8}$, then evaluate $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. E

OR

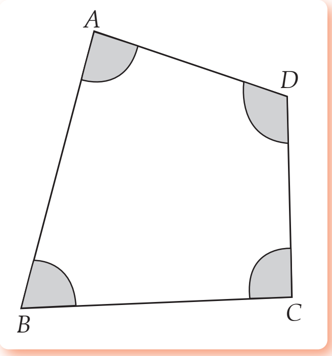
Find the value of θ for which the below statement is true. θ is an acute angle.
$\sqrt{3}\tan\theta - \cot 45° = 0$
Show your work. A

23. Find the area of segment AYB shown in the figure if the radius of the circle is 21 cm and $\angle AOB = 120°$ and $OM = \dfrac{21}{2}$ cm (use $\pi = \dfrac{22}{7}$). A



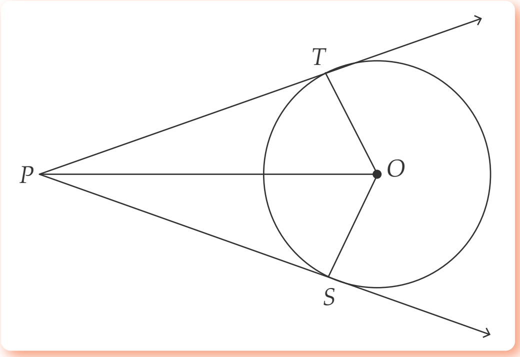
OR

In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C, and D of quadrilateral $ABCD$ as centres. Find the area of the shaded region. A



24. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

25. In the given figure, from point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120°$. Prove that $OP = 2\,PS$. U



Section – C

This section consists of 6 questions of 3 marks each.

26. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre. U

OR

If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$. AI A

27. Two real numbers c and d satisfy the following equations:

$2c - 3d = 7$

$4c + d = 1$

Find the product of c and d. Show your work. R

OR

3 bags and 4 pens together cost ₹ 257, whereas 4 bags and 3 pens together cost ₹324. Find the total cost of 1 bag and 10 pens. Ap

28. If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$; $A > B$. Then find the values of A and B. R

29. If α and β are the zeros of the quadratic polynomial $P(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$ and find zeros of $P(x)$. R

30. One card is drawn from a well–shuffled deck of 52 cards. Find the probability of getting Ap

(i) a face card.

(ii) a jack of hearts.

(iii) a queen of the black suit.

31. Find HCF and LCM of 404 and 96 and verify that $HCF \times LCM$ = product of the two given numbers. U

Section – D

This section consists of 4 questions of 5 marks each.

32. Solve: $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$, $x \neq 0, 2$. E

OR

In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects. C A

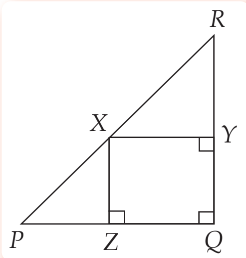
33. $(1, -1)$, $(0, 4)$ and $(-5, 3)$ are vertices of a triangle. Check whether it is a scalene triangle, an isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1, -1)$ at the midpoint of the opposite side. A

34. The ratio of the sums of first m and first n terms of an AP is $m^2:n^2$. Show that the ratio of its m^{th} and n^{th} term is $(2m - 1):(2n - 1)$. [R]

35. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium. [A]

OR

ΔPQR is right–angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ ar drawn. Prove that $XZ^2 = PZ \times ZQ$. [U]



Section – E

This section consists of 3 case-study-based questions of 4 marks each.

36. In a village, a group of people complained about an electric fault in their area. On their complaint, an electrician reached the village to repair the electric fault at a pole with a height of 5 m. She needs to reach a point 1.3 m below the top of the pole to undertake the repair work. She used a ladder, inclined at angle θ to the repair work. She used a ladder, inclined at angle θ to the horizontal such that cos θ = 0.5 to reach the required position. On the basis of the above information, answer the following: [Ap]

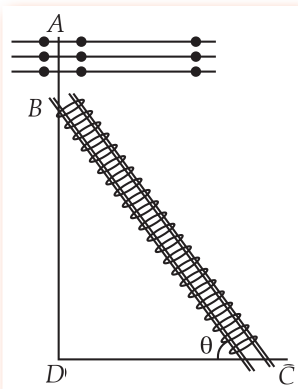
(i) Find the angle of elevation θ. **[1]**

(ii) Find the length of the ladder. **[1]**

(iii) How far from the foot of the pole should she place the foot of the ladder? **[2]**

OR

If the height of the pole and distance BD are double, then what will be the length of the ladder?



37. A stop watch was used to find the time that it took a group of students to run 100 m.

Time (in seconds)	0–20	20–40	40–60	60–80	80–100
No. of students	8	10	13	6	3

R

(i) Estimate the mean time taken by a student to finish the race. **[2]**

OR

(i) Find the median of the given data.

(ii) Find the sum of the lower limits of the median class and modal class. **[1]**

(iii) How many students finished the race within 1 minute? **[1]**

38. A mathematics teacher of a school took her 10^{th} standard students to show Gol Gumbaz. It was a part of their educational trip. The teacher had an interest in history as well. She narrated the facts of Gol Gumbaz to students. Gol Gumbaz is the tomb of King Muhammad Adil Shah of the Adil Shah Dynasty. The construction of the tomb, located in Vijaypura, Karnataka, India, was started in 1626 and completed in 1656. The teacher said in this monument one can find a combination of solid figures. She pointed out that there are cubical bases and some hemispherical shapes at the top. Ap

(i) What is the diagonal of the cubic portion of the Gol Gumbaz if one side of a cubical portion is 23 m? **[1]**

(ii) A block of Gol Gumbaz is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the shape is 2 cm. Find the volume of the block. **[2]**

OR

Find the total surface area of a hemispherical dome with a radius of 7 cm.

(iii) If two solid hemispheres of the same base radius r are joined together along their basis, then find the curved surface area of this new solid. **[1]**

SOLUTIONS

Sample Question Paper-5

MATHEMATICS STANDARD

Section – A

1. Option (B) is correct.

Explanation: We know that:

The positions of the flag form an AP as follows:

10, 15, 20, 25 …

We have:

First term, $a = 10$

Common difference, $d = 5$

Number of flags, $n = 20$

20th term of AP $= a_{20} = a + (20 - 1)d$

$\Rightarrow \quad a_{20} = 10 + 19 \times 5 = 10 + 95 = 105$

So the distance covered by a player to collect the 20th flag = 105 m

Total distance covered in one round = 105 + 105 = 210 m

2. Option (A) is correct.

Explanation: The product of a non-zero rational and an irrational number is always irrational.

3. Option (D) is correct.

Explanation:

CI	f	Cumulative frequency
0–5.5	26	26
5.5–11.5	20	46
11.5–17.5	30	76
17.5–23.5	16	92
23.5–29.5	32	124

$$N = 124 \Rightarrow \frac{N}{2} = \frac{124}{2} = 62$$

$\therefore \quad$ Median class = 11.5 – 17.5

Therefore, upper limit of median class = 17.5

4. Option (C) is correct.

Explanation:

Given:

$$\frac{r_1}{r_2} = \frac{3}{5}$$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} \quad (\because \text{ height of both cones are same})$$

$$= \frac{r_1^2}{r_2^2} = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25}$$

5. Option (B) is correct.

Explanation:

Let $P(x) = x^2 + (a + 1)x + b$

Given, 2 and –3 are zeroes of $P(x)$.

$\therefore \quad P(2) = (2)^2 + (a + 1)\,2 + b = 0$

$4 + 2a + 2 + b = 0$

$2a + b = -6 \quad$ ...(i)

And $P(-3) = (-3)^2 + (a+1)(-3) + b = 0$

$9 - 3a - 3 + b = 0$

$-3a + b = -6 \quad$ ...(ii)

Subtracting eq. (ii) from eq. (i), we get:

$$\begin{array}{r} 2a + b = -6 \\ -3a + b = -6 \\ + \quad - \quad + \\ \hline 5a = 0 \Rightarrow a = 0 \end{array}$$

Putting the value of a in eq (i), we get:

$2 \times 0 + b = -6$

$b = -6$

Hence, $\quad a = 0, b = -6$

6. Option (C) is correct.

Explanation:

$2x + ky = 1$ and $3x - 5y = 7$ are parallel.

Here, $a_1 = 2, \quad b_1 = k, \quad c_1 = 1$

$a_2 = 3, \quad b_2 = -5, \quad c_2 = 7$

We know that for parallel lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2}{3} = \frac{k}{-5} \neq \frac{1}{7}$$

$$\frac{2}{3} = \frac{k}{-5} \Rightarrow 3k = -10$$

$$\Rightarrow k = \frac{-10}{3}$$

7. Option (B) is correct.

Explanation: Given:

$2x^2 + kx + 2 = 0$

Comparing the above equation with:

$ax^2 + bx + c = 0,$

$a = 2, b = k$ and $c = 2$

Condition for equal roots is:

$D = 0$

i.e., $\quad b^2 - 4ac = 0$

Substituting the values of a, b and c, we get:

$k^2 - 4 \times 2 \times 2 = 0$

$\Rightarrow \quad k^2 - 16 = 0$

$\Rightarrow \quad [(k)^2 - (4)^2] = 0$

$\Rightarrow \quad (k + 4)(k - 4) = 0$

$\Rightarrow \quad k = 4$ or -4

8. **Option (C) is correct.**

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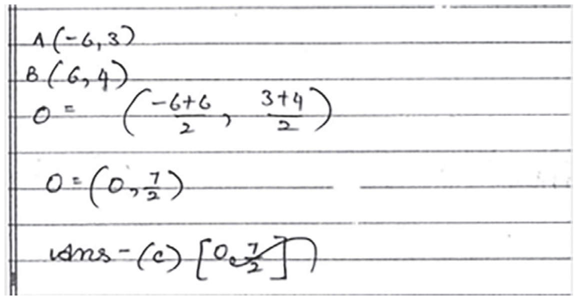
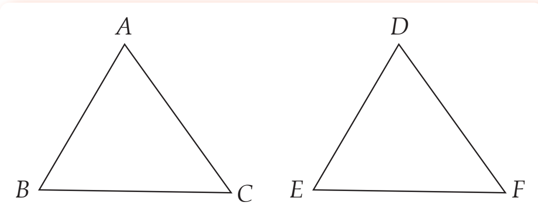

$A(-6,3)$
$B(6,4)$
$O = \left(\frac{-6+6}{2}, \frac{3+4}{2}\right)$
$O = \left(0, \frac{7}{2}\right)$
Ans - (c) $\left[0, \frac{7}{2}\right]$

9. **Option (B) is correct.**

Explanation: According to the definition of similarity of two triangles, 'Two triangles are similar when their corresponding angels are equal and the sides are in proportion.'



According to the question:

$$\angle F = \angle C \text{ and } \angle B = \angle E$$

Since, $AB = \frac{1}{2}DE$ (given)

Also, $\frac{AB}{DE} = \frac{1}{2}$

It means the triangles are similar but not congruent.

10. **Option (D) is correct.**

Explanation:

$$\cos(40°+A) = \sin 30°$$

$$\cos(40°+A) = \frac{1}{2}$$

$$\cos(40°+A) = \cos 60° \quad \left(\because \cos 60° = \frac{1}{2}\right)$$

$$40° + A = 60°$$

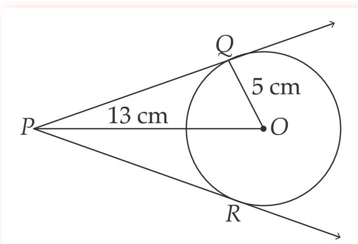
$$A = 60° - 40°$$

$$= 20°$$

11. **Option (C) is correct.**

Explanation:

We know that the tangent is perpendicular to the radius through the point of contact.



$\therefore \quad \angle OQP = 90°$

Now, in ΔPOQ:

$$OP^2 = PQ^2 + OQ^2$$

$$(13)^2 = PQ^2 + 5^2$$

$$\Rightarrow \quad PQ^2 = 169 - 25 = 144$$

$$\Rightarrow \quad PQ = 12 \text{ cm}$$

Hence, $PQ = PR = 12$ cm

12. **Option (C) is correct.**

Explanation: We know that:

Mode is the central tendency that tells the most frequent item in a dataset.

So mode helps in determining which shoe size to be ordered the most.

13. **Option (A) is correct.**

Explanation: Area of shaded region = area of sector with angles 60°, 80° and 40°

$$= \frac{60°}{360°} \times \pi r^2 + \frac{80°}{360°} \times \pi r^2 + \frac{40°}{360°} \times \pi r^2$$

$$= \frac{\pi r^2}{360°}[60° + 80° + 40°]$$

$$= \frac{\pi r^2}{360°} \times 180° = \frac{\pi r^2}{2}$$

$$= \frac{22}{7} \times \frac{1}{2} \times 7 \times 7 = 77 \text{ cm}^2$$

14. **Option (A) is correct.**

Explanation:

Given, $AP = 3.5$ cm, $PB = 7$ cm, $AQ = 3$ cm, $QC = 6$ cm and $PQ = 4.5$ cm

Here, $\frac{AP}{AB} = \frac{3.5}{3.5+7} = \frac{3.5}{10.5} = \frac{1}{3}$

And $\frac{AQ}{AC} = \frac{3}{3+6} = \frac{3}{9} = \frac{1}{3}$

In ΔABC and ΔAPQ, $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{1}{3}$

$$\angle A = \angle A \quad \text{(common)}$$

Therefore, $\Delta APQ \sim \Delta ABC$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 4.5 \times 3$$

$$= 13.5 \text{ cm}$$

15. **Option (C) is correct.**

Explanation: According to figure:

In the right ΔDBC, $\tan 45° = \frac{DC}{BC}$

$$\Rightarrow \quad 1 = \frac{45}{BC}$$

$$\Rightarrow \quad BC = 45 \text{ m}$$

In the right ΔDAC, $\tan 30° = \frac{DC}{AB+BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{45}{AB+45}$$

$$\Rightarrow \quad AB + 45 = 45\sqrt{3}$$

$$\Rightarrow \quad AB = 45(\sqrt{3}-1) \text{ m}$$

16. Option (D) is correct.

Explanation:

Given, $\sin 2A = \frac{1}{2}\tan^2 45°$

$= \frac{1}{2} \times 1 \quad (\because \tan 45° = 1)$

$\Rightarrow \sin 2A = \frac{1}{2}$

$\Rightarrow \sin 2A = \sin 30°$

$\therefore 2A = 30°$

$A = \frac{30}{2} = 15°$

17. Option (C) is correct.

Explanation: We have:

Total number of outcomes = 10

Multiples of 2 or 5 in 31 – 40 = {32, 34, 35, 36, 38, 40}

Number of favourable outcomes = 6

Required probability = $\frac{6}{10}$

18. Option (D) is correct.

Explanation: The sum of an interior angle of a polygon is $(n-2)180°$

So:

Each internal angle of a regular hexagon =120°

Area of 1 sector = $\frac{120°}{360°} \times \pi r^2$

$= \frac{1}{3}\pi r^2$

Area of 6 sectors = $6 \times \frac{1}{3}\pi r^2$

$= 2\pi r^2$

19. Option (A) is correct.

Explanation:

Weight (in kg) (x_i)	No. of workers (f_i)	$f_i x_i$
60	4	240
63	3	189
66	2	132
69	2	138
72	1	72
	12	771

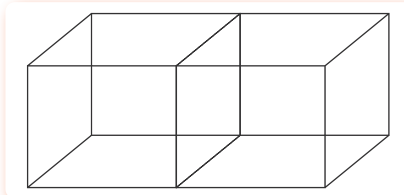
Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{771}{12} = 64.25$

Hence, both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

20. Option (D) is correct.

Explanation:

When two cubes of side 5 cm are joined end to end:



$\therefore l = 5 + 5 = 10$ cm, $b = 5$ cm, $h = 5$ cm

$\therefore$ Total surface area of cuboid = $2(lb + bh + hl)$

$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)$

$= 2(125) = 250 \text{ cm}^2$

So Assertion (A) is false, but Reason (R) is true.

Section – B

21. In ΔABP:

$DC \parallel AP$ (given)

$\therefore \frac{BD}{DA} = \frac{BC}{CP}$ (from BPT) ...(i)

In ΔABC,

$DE \parallel AC$ (given)

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (from BPT) ...(ii)

From equations (i) and (ii), we get:

$\frac{BE}{EC} = \frac{BC}{CP}$ **Hence, proved.**

22. Given that, $\cot\theta = \frac{7}{8}$

$\therefore \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$(\because (a-b)(a+b) = a^2 - b^2)$

$= \frac{\cos^2\theta}{\sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

$\frac{(1-\sin\theta)(1+\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{49}{64}$

OR

We have:

$\sqrt{3}\tan\theta - \cot 45° = 0$

$\sqrt{3}\tan\theta = \cot 45°$

$\sqrt{3}\tan\theta = 1$ (as $\cot 45° = 1$)

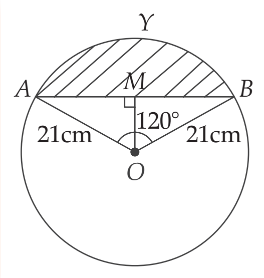
$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30°$

On comparing:

$\theta = 30°$ is the required acute angle.

23.



Given, $r = 21$ cm, $\theta = 120°$

Area of sector $OAYB = \frac{\theta}{360°} \times \pi r^2$

$= \frac{120}{360} \times \pi(21)^2$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21$

$= 462 \text{ cm}^2$

$\because$ $OM \perp AB$

$\therefore$ M is the midpoint of AB.

Now, in ΔOMA, $AM^2 = OA^2 - OM^2$

$= (21)^2 - \left(\frac{21}{2}\right)^2$

$= 441 - \frac{441}{4}$

$= 441\left(1 - \frac{1}{4}\right) = 441\left(\frac{3}{4}\right)$

$\therefore$ $AM = 21\frac{\sqrt{3}}{2}$ cm

So $AB = 2 \times AM = \frac{2 \times 21\sqrt{3}}{2}$

$= 21\sqrt{3}$ cm

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OM \times AB$

$= \frac{1}{2} \times \frac{21}{2} \times 21\sqrt{3}$

$= \frac{441\sqrt{3}}{4} \text{cm}^2$

$\therefore$ Area of segment = area of sector – area of ΔOAB

$= 462 - \frac{441\sqrt{3}}{4}$

$= \frac{21}{4}\left(88 - 21\sqrt{3}\right)\text{cm}^2$

Commonly Made Error

- Some students don't know how to find the area of a segment of a circle.

Answering Tip

- Students must practice more questions and learn how to find the area of a segment of a circle.

OR

Let the measure of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ be θ_1, θ_2, θ_3 and θ_4 respectively.

Require area = area of the sector with centre A
+ area of the sector with centre B
+ area of the sector with centre C
+ area of the sector with centre D

$= \frac{\theta_1}{360°} \times \pi \times 7^2 + \frac{\theta_2}{360°} \times \pi \times 7^2 + \frac{\theta_3}{360°} \times \pi \times 7^2 + \frac{\theta_4}{360°} \times \pi \times 7^2$

$= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360°} \times \pi \times 7^2$

$= \frac{(360°)}{360°} \times \frac{22}{7} \times 7 \times 7$

(by angle sum property of quadrilateral)

$= 154 \text{ cm}^2$

24. We know that composite numbers are numbers that have more than two factors.

Now, $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1)$

$= 13(77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6 \times 1$

$= 13 \times 13 \times 2 \times 3 \times 1$

The given number has 2, 3, 13 and 1 as its factors.

So it is a composite number.

And $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1{,}008 + 1)$

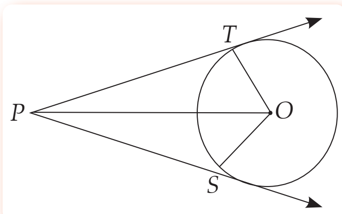
$= 5 \times 1{,}009 \times 1$

1,009 cannot be factorised further.

Therefore, the given expression has 5, 1,009 and 1 as its factors.

Hence, it is a composite number.

25. $\because$ Tangents drawn from an external point are equal in length.



$\therefore$ $PT = PS$

In ΔTPO and ΔSPO:

$\angle OTP = \angle OSP = 90°$

($\because$ tangent is perpendicular to the radius through the point of contact)

$OT = OS$ (each radii)

$OP = OP$ (common)

$\therefore$ $\Delta TPO \cong \Delta SPO$

(by SAS congruence rule)

$\therefore$ $\angle TPO = \angle SPO$ (by $CPCT$)

$\angle TPO = \frac{1}{2}\angle SPT = \frac{1}{2} \times 120°$

$= 60°$

In ΔSPO, $\cos 60° = \frac{PS}{OP}$

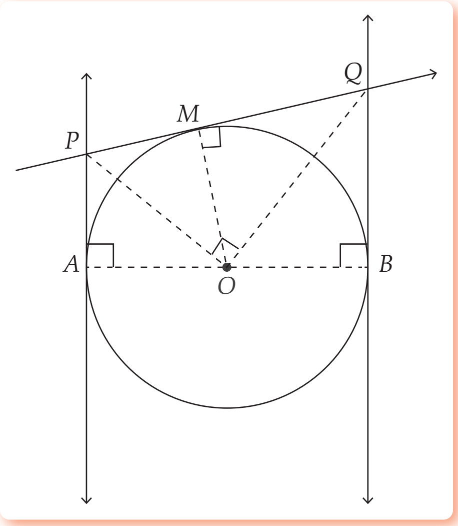
$$\frac{1}{2} = \frac{PS}{OP}$$

$\Rightarrow$ $OP = 2PS$ **Hence, proved.**

Section – C

26. Let a circle with centre O has two parallel tangents PA and QB through A and B, respectively, at the end of diameter.

Let tangent through M intersect the parallel tangents at P and Q.



$\because$ $\angle A = 90° = \angle B$

(tangent is perpendicular to radius through the point of contact)

$\therefore$ $\angle A + \angle B = 180°$ ...(i)

$\because$ $ABQP$ is a quadrilateral.

$\therefore$ $\angle A + \angle B + \angle Q + \angle P = 360°$

$\angle Q + \angle P = 360° - 180°$ (from (i))

$\angle Q + \angle P = 180°$ ...(ii)

Now, $\angle APO = \angle OPQ = \frac{1}{2}\angle P$ ...(iii)

(a line drawn from centre to point where external tangents bisect the angle made by tangents at that point)

Similarly, $\angle BQO = \angle PQO = \frac{1}{2}\angle Q$ ...(iv)

Using (iii) and (iv) in eq. (ii), we get:

$2\angle OPQ + 2\angle PQO = 180$

$\angle OPQ + \angle PQO = \frac{180}{2} = 90°$ ...(v)

In ΔOPQ:

$\angle OPQ + \angle PQO + \angle POQ = 180°$

$90° + \angle POQ = 180°$

$\angle POQ = 180–90°$

$\angle POQ = 90°$ **Hence, proved.**

Commonly Made Error

- Some students don't make a proper figure according to given conditions. Thus, they are not able to prove the required result.

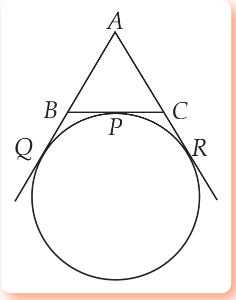
Answering Tip

- Students must make a well-labelled figure to prove the required result and practice more questions.

OR

$BC + AC + AB = (BP + PC) + (AR - RC) + (AQ - BQ)$

$= AQ + AR - BQ + PC - CR$



$\because$ From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

From the point C, $CP = RC$

$\therefore$ Permitter of ΔABC:

$AB + BC + AC = 2AQ - BQ + BQ + CR - CR$

$\Rightarrow$ $= 2AQ$

$\Rightarrow$ $AQ = \frac{1}{2}(BC + CA + AB)$

Hence, proved.

27. Given that:

$2c - 3d = 7$ ...(i)

$4c + d = 1$ ...(ii)

Multiplying eq. (i) by 2 and subtracting from eq. (ii), we get:

$4c + d = 1$

$4c - 6d = 14$

$-\quad +\quad -$

$7d = -13$

$d = -\frac{13}{7}$

Putting the value of d in eq. (ii) we get:

$\Rightarrow$ $4c + \frac{-13}{7} = 1$

$\Rightarrow$ $4c = \frac{13}{7} + 1$

$\Rightarrow$ $4c = \frac{20}{7}$

$\Rightarrow \qquad c = \frac{5}{7}$

Now, the product of c and d is:

$$\Rightarrow \qquad c \times d = \frac{5}{7} \times \left(-\frac{13}{7}\right) = -\frac{65}{49}$$

OR

Let the cost of 1 bag and 1 pen be x and y, respectively. Then:

$$3x + 4y = 257 \qquad ...(i)$$
$$4x + 3y = 324 \qquad ...(ii)$$

Multiplying eq. (i) by 4 and eq. (ii) by 3 and then subtracting eq. (ii) from eq. (i), we get:

$$12x + 16y = 1028$$
$$12x + 9y = 972$$
$$- \quad - \quad -$$
$$7y = 56$$
$$y = \frac{56}{7} = 8$$

Putting the value of y in eq(i), we get:

$$3x + 4(8) = 257$$
$$3x = 257 - 32 = 225$$
$$x = \frac{225}{3} = 75$$

Hence, cost of 1 bag = ₹75

Cost of 1 pen = ₹8

Now,

Total cost of 1 bag and 10 pens

$= 75 + 10 \times 8$

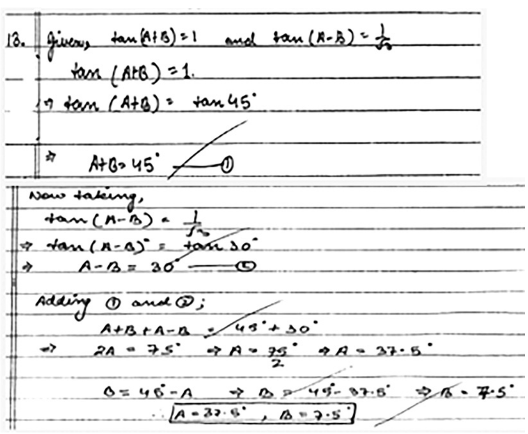
$= 75 + 80$

$= 155$

Hence, the total cost is ₹ 155.

28.

Topper Answer, 2019


13. Given, tan (A+B) = 1 and tan (A−B) = 1/√3

tan (A+B) = 1.

⇒ tan (A+B) = tan 45°

⇒ A+B = 45° —— ①

Now taking,

tan (A−B) = 1/√3

⇒ tan (A−B)° = tan 30°

⇒ A−B = 30° —— ②

Adding ① and ②;

A+B+A−B = 45° + 30°

⇒ 2A = 75° ⇒ A = 75°/2 ⇒ A = 37.5°

B = 45° − A ⇒ B = 45° − 37.5° ⇒ B = 7.5°

∴ A = 37.5°, B = 7.5°

29. $P(x) = 4x^2 - 5x - 1$

$\because$ α and β are zeros of $P(x)$.

$$\therefore \qquad \alpha + \beta = \frac{-b}{a} = \frac{5}{4}$$

$$\text{and} \qquad \alpha\beta = \frac{c}{a} = \frac{-1}{4}$$

$$\therefore \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$$
$$= \frac{-1}{4} \times \frac{5}{4} = \frac{-5}{16}$$

Now, $\because P(x) = 4x^2 - 5x - 1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-1)}}{2 \times 4}$$
$$= \frac{5 \pm \sqrt{25 + 16}}{8} = \frac{5 \pm \sqrt{41}}{8}$$
$$\therefore \qquad \alpha = \frac{5 + \sqrt{41}}{8}, \quad \beta = \frac{5 - \sqrt{41}}{8}$$

30. **(i)** Total cards = 52

Face cards = 4 kings + 4 queen + 4 jack

= 12

$$\therefore P\text{ (face card)} = \frac{\text{No. of face cards}}{\text{Total cards}}$$
$$= \frac{12}{52} = \frac{3}{13}$$

(ii) P (a Jack of heart) $= \frac{1}{52}$

(iii) P (a queen of black suit) $= \frac{2}{52} = \frac{1}{26}$

Commonly Made Error

- Some students don't write the final probability in simplified form leading to deduction of marks.

Answering Tip

- Students must represent the final result of probability in simplified form to get full marks.

31. Since $\qquad 404 = 2 \times 2 \times 101 = 2^2 \times 101$

and $\qquad 96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$= 2^5 \times 3$

$\therefore$ HCF of 404 and 96 $= 2^2 = 4$

LCM of 404 and 96 $= 101 \times 2^5 \times 3 = 9{,}696$

HCF × LCM $= 4 \times 9{,}696 \times 38{,}784$

Also, $\qquad 404 \times 96 = 38{,}784$

Hence, HCF × LCM = product of 404 and 96

Hence, proved.

Section – D

32.

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

$$\frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\frac{x^2 + 3x - (x - 2 - x^2 + 2x)}{x^2 - 2x} = \frac{17}{4}$$

$$4\left[x^2 + 3x - 3x + x^2 + 2\right] = 17\left(x^2 - 2x\right)$$

$$8x^2 + 8 = 17x^2 - 34x$$

$$9x^2 - 34x - 8 = 0$$
$$9x^2 - 36x + 2x - 8 = 0$$
$$9x(x - 4) + 2(x - 4) = 0$$
$$(x - 4)(9x + 2) = 0$$
$$x = 4, x = \frac{-2}{9}$$

OR

Let the marks in Hindi be x and the marks in English be y.

According to the question:

$$x + y = 30$$
$$\Rightarrow \quad y = 30 - x \quad \text{...(i)}$$

If he had got 2 marks more in Hindi, then his marks would $b = x + 2$ and if he had 3 marks less in English, then his marks would be $= y - 3$.

According to the question:

$$(x + 2)(y - 3) = 210$$
$$\Rightarrow \quad (x + 2)(30 - x - 3) = 210 \quad \text{(from eq. (i))}$$
$$\Rightarrow \quad (x + 2)(27 - x) = 210$$
$$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210$$
$$\Rightarrow \quad -x^2 + 25x - 156 = 0$$
$$\Rightarrow \quad x^2 - 25x + 156 = 0$$
$$\Rightarrow \quad x^2 - 13x - 12x + 156 = 0$$
$$\Rightarrow \quad x(x - 13) - 12(x - 13) = 0$$
$$\Rightarrow \quad (x - 12)(x - 13) = 0$$

$\Rightarrow$ Either $x = 12$ or $x = 13$

When $x = 12$,

then $y = 30 - 12 = 18$.

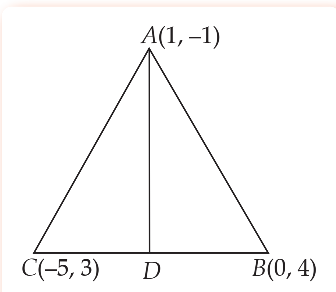
When $x = 13$,

then $x = 30 - 13 = 17$.

Hence, the marks in Hindi = 12 and the marks in English = 18

Or the marks in Hindi = 13 and the marks in English = 17

33.



Let the vertices of ΔABC be $A(1, -1)$, $B(0, 4)$ and $C(-5, 3)$.

$\therefore$ Using the distance formula:

$$AB = \left|\sqrt{(1-0)^2 + (-1-4)^2}\right|$$
$$= \left|\sqrt{1+5^2}\right|$$
$$= \sqrt{26} \text{ units}$$
$$BC = \left|\sqrt{(-5-0)^2 + (3-4)^2}\right|$$
$$= \left|\sqrt{25+1}\right| = \sqrt{26} \text{ units}$$
$$AC = \left|\sqrt{(-5-1)^2 + (3+1)^2}\right|$$
$$= \left|\sqrt{36+16}\right| = \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

Or $\quad AB = BC \neq AC$

Or ΔABC is isosceles.

Now, using midpoint formula, the co-ordinates of midpoint of BC are:

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$
$$y = \frac{3+4}{2} = -\frac{7}{2}$$

Or, $\quad D(x, y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$

$\therefore$ Length of median, AD

$$= \left|\sqrt{\left(-\frac{5}{2}, \frac{7}{2}^{-1}\right)^2 + \left(\frac{7}{2}+1\right)^2}\right|$$
$$= \left|\sqrt{\left(\frac{-7}{2}\right)^2 + \left(\frac{9}{2}\right)^2}\right|$$
$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ units}$$

$\therefore$ Length of median AD is $\frac{\sqrt{130}}{2}$ units.

34.

$$S_m = \frac{m}{2}\left[2a + (m-1)d\right]$$

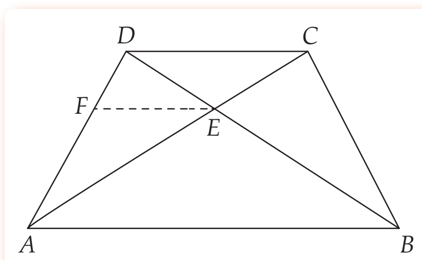
and $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$\frac{S_m}{S_n} = \frac{m\left(2a + (m-1)d\right)}{n\left(2a + (n-1)d\right)}$$
$$\frac{m}{n} = \frac{2a + (m-1)d}{2a + (n-1)d}$$
$$m(2a + (n-1)d) = n(2a + (m-1)d)$$
$$2am + m(n-1)d = 2an + n(m-1)d$$
$$2a(m-n) + (mn - m - mn + n)d = 0$$
$$2a(m-n) = (m-n)d$$
$$2a = d \quad (\because m \neq n)$$

Now, $\frac{a_m}{a_n} = \frac{a + (m-1)d}{a + (n-1)d}$

$$= \frac{a + (m-1)2a}{a + (n-1)2a} = \frac{1 + 2m - 2}{1 + 2n - 2}$$
$$\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$$

35.



Given: A quadrilateral $ABCD$ whose diagonals AC and BD intersect at E. Such that:

$$\frac{DE}{EB} = \frac{CE}{EA}$$

To prove: $ABCD$ is a trapezium, i.e., $AB \parallel DC$.

Construction: Draw $EF \parallel BA$, meeting AD at F.

Proof: In ΔABD, $FE \parallel AB$.

$\therefore \quad \frac{DE}{EB} = \frac{DF}{FA}$ ...(i) (by Thale's theorem)

But $\quad \frac{DE}{EB} = \frac{CE}{EA}$ ...(ii) (Given)

$\frac{DF}{FA} = \frac{CE}{EA}$ (from eq. (i) and eq. (ii))

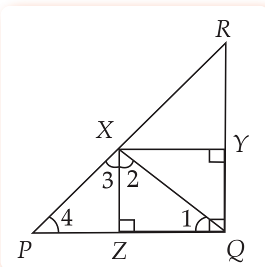
$\therefore \quad FE \parallel DC$ (converse of Thale's theorem)

But $\quad FE \parallel AB$

$\therefore \quad AB \parallel DC$, and hence $ABCD$ is a trapezium.

Hence, proved.

OR



Here, $RQ \perp PQ$ and $XZ \perp PQ$

or $\quad XZ \parallel YQ$

$\therefore$ Similarly, $\quad XY \parallel ZQ$

$XYQZ$ is rectangle. $\quad (\because \angle PQR = 90°)$

In ΔXZQ, $\quad \angle 1 + \angle 2 = 90°$ ...(i)

In ΔPZX, $\quad \angle 3 + \angle 4 = 90°$ ...(ii)

$XQ \perp PR$

Or, $\quad \angle 2 + \angle 3 = 90°$ ...(iii)

By eqs. (i) and (iii), we get:

$$\angle 1 = \angle 3$$

By eqs. (ii) and (iii), we get:

$$\angle 2 = \angle 4$$

$\therefore \quad \Delta PZX \sim \Delta XZQ$ (AA similarity)

$\therefore \quad \frac{PZ}{XZ} = \frac{XZ}{ZQ}$

Thus, $\quad XZ^2 = PZ \times ZQ$

Hence, proved.

Section – E

36. **(i)** Given $\quad \cos\theta = 0.5 = \frac{1}{2}$

$\cos\theta = \cos 60°$

$\theta = 60°$

(ii) $\because \quad BD = AD - AB$

$= (5 - 1.3)$

$= 3.7$ m

Now, in ΔBDC:

$$\sin 60° = \frac{BD}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.7}{BC}$$

$$\Rightarrow \quad BC = \frac{3.7 \times 2}{\sqrt{3}}$$

$$BC = \frac{7.4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{7.4\sqrt{3}}{3}$$

$$= 2.46\sqrt{3}$$

$$BC = 4.28 \text{ m}$$

(iii) In ΔBDC:

$$\cot 60° = \frac{DC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{DC}{3.7}$$

$$DC = \frac{3.7}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} = \frac{3.7}{3}\sqrt{3}$$

$$= 1.233\sqrt{3}$$

$$= 2.14 \text{ m}$$

OR

Given, BD is doubled.

So $\quad$ new $BD = 3.7 \times 2 = 1.4$ m

$\therefore$ In ΔBDC, $\quad \sin 60° = \frac{BD}{BC}$

$$\frac{\sqrt{3}}{2} = \frac{7.4}{BC}$$

$$BC = \frac{7.4 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

New length of ladder $= \frac{14.8\sqrt{3}}{3} = 8.56$ m

37. **(i)**

Time (in second)	Midpoint (x_i)	No. of students (f_i)	$f_i x_i$
0–20	10	8	80
20–40	30	10	300
40–60	50	13	650
60–80	70	6	420
80–100	90	3	270
		40	$\Sigma f_i x_i = 1{,}720$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1{,}720}{40} = 4.3$$

OR

Time (in second)	Midpoint (x_i)	No. of students (f_i)	Cumulative frequency
0–20	10	8	8
20–40	30	10	18
40–60	50	13	31
60–80	70	6	37
80–100	90	3	40
		40	

$$N = 40$$

$$\Rightarrow \quad \frac{N}{2} = 20$$

$\therefore$ Median class = 40 – 60

Lower limit of median class (c) = 40

Frequency (f) = 13

CF = 18, h = 20

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 40 + \frac{20 - 18}{13} \times 20$$

$$= 40 + \frac{40}{13} = 43.07$$

(ii) $\because$ Median class= 40 – 60

$\therefore$ Lower limit of median class = 40

From the given data, the maximum frequency is 13 which belongs to 40–60.

$\therefore$ Modal class = 40 – 60

Lower limit of modal class = 40

$\therefore$ Required sum = 40 + 40

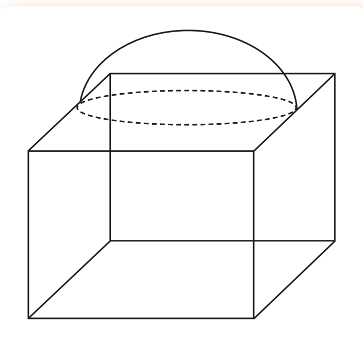
= 80

(iii) No. of students those who finished the race within 1 min

= 8 + 10 + 13

= 31

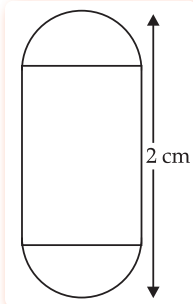
38. (i) $\because$ Diagonal of cubic portion = $a\sqrt{3}$



Given, $a = 23$ m

$\therefore$ Diagonal = $23\sqrt{3}$ m

(ii) Volume of a block = volume of cylinder + 2(volume of hemisphere)



Given, $r = \frac{0.5}{2} = 0.25$ cm

Total length = 2 cm

$\therefore$ Height of cylinder (h) = 2 – 0.5

= 1.5 cm

$$\therefore \quad \text{Volume of a block} = \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2\left[h + \frac{4}{3}r\right]$$

$$= 8.14 \times (0.25)^2\left[1.5 + \frac{4}{3} \times 0.25\right]$$

$$= 3.14 \times (0.25)^2 \times \frac{5.5}{3}$$

$$= 0.36 \text{ cm}^3$$

OR

Total surface area of hemispherical dome = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times 7 \times 7$$

$$= 462 \text{ cm}^2$$

(iii) If two hemispheres of the same base radius (r) are joined together along their bases, then:

Curved surface area of new solid = $2\pi r^2 + 2\pi r^2$

= $4\pi r^2$

■■■